Influence of zonal flows on unstable drift modes in ETG turbulence

V M Lashkin[1], **Yu A Zaliznyak**[1] **and A I Yakimenko**[1,2]

[1]Institute for Nuclear Research, Kiev 03680, Ukraine

[2]Department of Physics, Taras Shevchenko National University, Kiev 03022, Ukraine

E-mail: `vlashkin@kinr.kiev.ua`

Abstract. The linear instability of the electron temperature gradient (ETG) driven modes in the presence of zonal flows is investigated. Random and deterministic *cos* -like profiles of the zonal flow are considered. It is shown that the presence of shearing by zonal flows can stabilize the linear instability of ETG drift modes.

PACS numbers: 52.35.-g, 52.30.-q, 52.55.Fa, 52.35.Kt

1. Introduction

The electron temperature gradient (ETG) driven mode [1, 2, 3] is often considered as a possible candidate for the explanation of electron thermal transport through internal transport barriers when ion temperature gradient (ITG) turbulent fluctuations are suppressed by $\mathbf{E}\times\mathbf{B}$ shear flow. At the same time, small scale ETG fluctuations have the typical spatial scale of the order of electron gyroradius ρ_e, and are less susceptible to quenching by shearing [4, 5, 6]. It is widely thought that drift-wave-type turbulence can excite zonal flows which are associated with azimuthally symmetric band-like shear flows that depend only on the radial coordinate [7, 8]. Zonal flows play a crucial role in regulating drift-wave turbulence and transport in tokamaks. It is now quite clear that zonal flows are generated by modulational instability of drift waves [9, 10, 11, 12, 13]. For ETG driven turbulence, the excitation of zonal flow was considered in [14] (the case of broad turbulent spectrum) and [15] (the four-wave coupling scheme).

It is well known that the presence of shear flow give rise not only to instability of the sheared layer (Kelvin–Helmholtz instability), but also to stabilization of other instabilities (ITG driven modes, resistive interchange modes etc.) [7, 16, 17, 18, 19]. Up to now, the stabilizing effect on the linear instabilities has been considered only for the case of mean smooth flows. Note, that zonal flow shearing differs from that for mean flow shearing on account of the complexity of the flow pattern. In contrast to smooth, static mean flows, the zonal flow patterns can be expected to have finite correlation time and complex, possibly random, spatial structure [7, 20].

In the present work we consider the influence of zonal flow on the linear stability of ETG drift modes. We show that the presence of random shearing by zonal flows strongly affects the linear stability of ETG modes and has a stabilizing effect. If the mean square amplitude of zonal flow exceeds some critical value, the linear instability of ETG modes is suppressed for all poloidal wave numbers k_y.

2. Basic equations

Assuming a slab two-dimensional geometry, charge quasineutrality and the adiabatic ion responce, we consider the following simplified model describing curvature driven ETG turbulence and including viscosity and thermal diffusivity [5]

$$\left(\frac{\partial}{\partial t} + \mathbf{v_E}\cdot\nabla\right)(\varphi - \Delta_\perp\varphi) + \frac{\partial}{\partial y}(\varphi + p) + \nu\Delta^2\varphi = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v_E}\cdot\nabla\right)p - r\frac{\partial}{\partial y}\varphi - \chi\Delta p = 0, \tag{2}$$

where $\mathbf{v_E} = [\hat{\mathbf{z}}\times\nabla\varphi]$ is $\mathbf{E}\times\mathbf{B}$ drift velocity, φ and P are the normalized electrostatic potential and plasma pressure respectively, and $\{A,B\} = \partial_x A\partial_y B - \partial_y A\partial_x B$ is the Jakobian. The variables in equations (1) and (2) have been rescaled as follows

$$r = \frac{\epsilon_B\epsilon_{*e}}{\epsilon_{*i}^2},$$

$$\varphi = \frac{1}{\epsilon_{*i}}\frac{e\phi}{T_i}, \qquad P = \frac{\epsilon_B}{\epsilon_{*i}^2}\frac{P}{P_{i0}},$$

$$x = \frac{x'}{\rho_s\sqrt{\tau}}, \qquad y = \frac{y'}{\rho_s\sqrt{\tau}}, \qquad t = \epsilon_{*i}\omega_{Bi}t',$$

x', y' and t' being the original physical coordinates (with x' the poloidal and y' the radial coordinate),

$$\epsilon_{*i} = \frac{\rho_s\sqrt{\tau}}{L_n}, \qquad \epsilon_B = \frac{\rho_s\sqrt{\tau}}{L_B}, \qquad \epsilon_{*e} = \frac{\rho_s\sqrt{\tau}}{L_p},$$

where L_n, L_B and L_p are the background gradient scales for the density, magnetic field and pressure respectively, ρ_s is the ion gyroradius calculated at the electron temperature T_e, and $\tau = T_i/T_e$. The effect of zonal flow can be included by assuming the $\mathbf{E}\times\mathbf{B}$ drift velocity of

$$\mathbf{v_E} = v(x)\hat{\mathbf{y}} + [\hat{\mathbf{z}}\times\nabla\varphi], \tag{3}$$

where the first term accounts for zonal flow. Neglecting nonlinear terms in equations (1) and (2), taking into account equation (3) and representing

$$\varphi(x,y,t) = \Phi(x)\exp(ik_y y - i\omega t),$$

$$p(x,y,t) = P(x)\exp(ik_y y - i\omega t),$$

one can obtain

$$(-i\omega + ik_y v)(1 + k_y^2 - \frac{d^2}{dx^2})\Phi + ik_y(\Phi + P) + \nu(k_y^4 + \frac{d^4}{dx^4} - 2k_y^2\frac{d^2}{dx^2})\Phi = 0, \tag{4}$$

$$(-i\omega + ik_y v)P - ik_y r\Phi + \chi(-k_y^2 + \frac{d^2}{dx^2})P = 0. \tag{5}$$

In the inviscid limit and absence of zonal flow, after taking $\Phi(x), P(x) \sim \exp(ik_x x)$, equations (4) and (5) give the dispersion relation for ETG modes

$$\omega_{1,2} = \frac{k_y}{2\,(k^2+1)}\left[1 \pm \sqrt{1 - 4r\,(k^2+1)}\right], \tag{6}$$

where $k^2 = k_x^2 + k_y^2$, and the plus sign describes the drift waves dispersion, while the minus sign corresponds to the dispersion of the convective cells. Equation (6) predicts instability with the growth rate

$$\gamma = \frac{k_y}{2(k^2+1)}\sqrt{4r(k^2+1) - 1}, \tag{7}$$

if $4r\,(k^2+1) \geq 1$.

3. ETG drift modes in the presence of zonal flow

We assume that the profile of zonal flow $v(x)$ is the random function and $v(x)$ is a zero mean, $\langle v(x)\rangle = 0$, real homogeneous Gaussian process with correlation function

$$\langle v(x)v(x')\rangle = D(x - x'), \tag{8}$$

where $\langle \ldots \rangle$ means statistical averaging. It is assumed, that the intensity of the noise is small, $D(x) \ll 1$. When neglecting v^2 term, and in the inviscid limit, equations (4) and (5) reduce to the equation

$$\omega^2\left(\frac{d^2}{dx^2} - k_y^2 - 1\right)\Phi + \omega\left\{k_y - 2k_y v\left(\frac{d^2}{dx^2} - k_y^2 - 1\right)\right\}\Phi - k_y^2(v + r)\Phi = 0, \tag{9}$$

which after the Fourier transforming $(d/dx \to -iq)$ can be rewritten as

$$G_0^{-1}(p)\Phi(p) + \int k_y[k_y - 2\omega(q_2^2 + k_y^2 + 1)]v(q_1)\Phi(\omega, q_2, k_y)\delta(q - q_1 - q_2)dq_1dq_2 = 0, \tag{10}$$

where $p \equiv (\omega, q, k_y)$ and

$$G_0^{-1}(p) = \omega^2(q^2 + k_y^2 + 1) - \omega k_y + k_y^2 r. \tag{11}$$

To avoid the cumbersome expressions, we write equation (10) in the symbolic form

$$G_0^{-1}(p)\Phi(p) + \gamma(p, p_1, p_2)v(p_1)\Phi(p_2) = 0, \tag{12}$$

where

$$\gamma(p, p_1, p_2) = \varkappa(p_1, p_2)\delta(p - p_1 - p_2), \tag{13}$$

and integration over repeated indices is assumed, as usual. Introducing a source $\eta(p)$ in the right hand side of equation (12) and taking the functional derivative $\delta/\delta\eta(p')$, we have

$$G_0^{-1}(p)G(p, p') + \gamma(p, p_1, p_2)v(p_1)G(p_2, p') = \delta(p - p'), \tag{14}$$

where $G(p, p') = \delta\Phi(p)/\delta\eta(p')$ is the Green function, and we have taken into account that $\delta\eta(p)/\delta\eta(p') = \delta(p - p')$. The function $G_0(p, p')$ is a free (i. e. in the absence of the noise v) Green function. Representing the Green function as a sum of the average and fluctuating parts

$$G(p, p') = \langle G(p, p') \rangle + \tilde{G}(p, p'), \tag{15}$$

substituting it into equation (14) and averaging, one can obtain

$$G_0^{-1}(p)\langle G(p, p')\rangle + \gamma(p, p_1, p_2)\langle v(p_1)\tilde{G}(p_2, p')\rangle = \delta(p - p'), \tag{16}$$

Substracting equation (16) from equation (14), we get

$$\begin{aligned} &G_0^{-1}(p)\tilde{G}(p, p') + \gamma(p, p_1, p_2)v(p_1)\langle G(p_2, p')\rangle \\ &+\gamma(p, p_1, p_2)[v(p_1)\tilde{G}(p_2, p') - \langle v(p_1)\tilde{G}(p_2, p')\rangle] = 0. \end{aligned} \tag{17}$$

In the Bourrett approximation [21], which is justified when the intensity of the noise is small enough, we can neglect the term $v\tilde{G} - \langle v\tilde{G}\rangle$ and get for the fluctuating part of the Green function

$$\tilde{G}(p, p') = -G_0(p)\gamma(p, p_1, p_2)v(p_1)\langle G(p_2, p')\rangle. \tag{18}$$

Inserting this expression into equation (16), we obtain

$$G_0^{-1}(p)\langle G(p, p')\rangle - \gamma(p, p_1, p_2)\gamma(p_1', p_2', p_2)G_0(p_2)\langle v(p_1)v(p_1')\rangle\langle G(p_2', p')\rangle = \delta(p - p'). \tag{19}$$

As follows from equation (8), in the wave number domain the correlator has the form $\langle v(p)v(p')\rangle = I(p)\delta(p + p')$. Due to homogeneity of the random process, the Green function in equation (19) has the structure $\langle G(p, p')\rangle = G(p)\delta(p - p')$. Making use of this and equation (13), one can perform some integrations in equation (19) and finally we get

$$G(p) = \frac{1}{G_0^{-1}(p) - \varkappa(p - p_1, p_1)\varkappa(p_1 - p, p)G_0(p_1)I(p - p_1)}. \tag{20}$$

The poles of the Green function $G(p)$ determine the spectrum of elementary excitations and the corresponding dispersion relation is

$$G_0^{-1}(p) - \varkappa(p - p_1, p_1)\varkappa(p_1 - p, p)G_0(p_1)I(p - p_1) = 0. \tag{21}$$

In the absence of zonal flow we have $G_0^{-1}(p) = 0$ and recover the previous result (6) with $k_x \equiv q$. In what follows we consider the case when the profile of the zonal flow is described by the random function $v(x)$ which has the form

$$v(x) = v_0 \cos(q_0 x + \vartheta), \tag{22}$$

where the random amplitude v_0 is a zero mean, normally distributed value with variance σ^2, and the random phase ϑ is uniformly distributed between 0 and 2π. The correlation function (8) of such a process is $D(x) = (\sigma^2/2)\cos(q_0 x)$ or, in the wave number domain

$$I(q) = \frac{\sigma^2}{4}[\delta(q - q_0) + \delta(q + q_0)]. \tag{23}$$

In this case the noise has an infinite correlation length and is concentrated at the wave number q_0 so that the characteristic scale length of the zonal flow is of the order $\sim q_0^{-1}$.

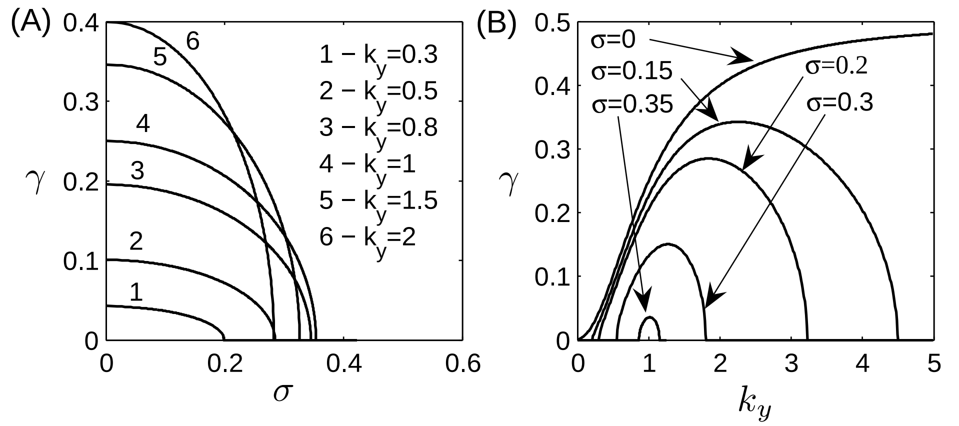

Figure 1. (A) The instability growth rate as a function of the root mean square zonal flow amplitude σ (for different values of the poloidal wave number k_y). (B) The growth rate as a function of the poloidal wave number k_y (for different values of σ).

As follows from eqution (7), in the absence of zonal flow the maximum growth rate for the fixed k_y is achieved at $k_x = 0$. Thus, to simplify calculations, we will consider the influence of zonal flow on the most unstable modes and put $q = 0$. Then, substituting equation (23) into equation (21) one can obtain the dispersion relation

$$c_1\omega^4 - c_2\omega^3 + c_3\omega^2 - c_4\omega + c_5 = 0, \tag{24}$$

where

$$c_1 = (k_y^2 + 1)(q_0^2 + k_y^2 + 1), \tag{25}$$

$$c_2 = k_y(q_0^2 + 2k_y^2 + 2), \tag{26}$$

$$c_3 = k_y^2(q_0^2 + k_y^2 + 1)[r - 2\sigma^2(k_y^2 + 1)] + k_y^2[1 + r(k_y^2 + 1)], \tag{27}$$

$$c_4 = 2k_y^3 r - k_y^3\sigma^2(q_0^2 + 2k_y^2 + 2), \tag{28}$$

$$c_5 = k_y^4\left(r^2 - \frac{\sigma^2}{2}\right). \tag{29}$$

Figure 1(a) shows the growth rate γ of the ETG driven mode as a function of the root mean square zonal flow amplitude σ for different values of the poloidal wave number k_y. The characteristic wave number of zonal flow used in the calculations is $q_0 = 0.1$. The parameter r has been fixed at $r = 0.25$ so that, as follows from equation (7), in the absence of random shearing there is a linear instability for all $k_y \neq 0$. It is seen that the growth rate decreases with increasing σ and vanishes above some value of σ which depends on k_y. Thus, zonal flow shearing stabilizes the instability of ETG modes. In figure 2(b) we plot the dependence of the growth rate on the poloidal wave number k_y for different values of the root mean square zonal flow amplitude σ. In the absence of zonal flow ($\sigma = 0$) the growth rate γ increases with increasing k_y and saturates at the level $\gamma_{max} = \sqrt{r}$. The presence of shearing ($\sigma \neq 0$) changes the situation drastically. The reduced growth rate initially increases as a function of k_y from some $k_{y,1}$ and then decreases, becoming zero at some $k_{y,2}$. For not too large σ, the linear instability is

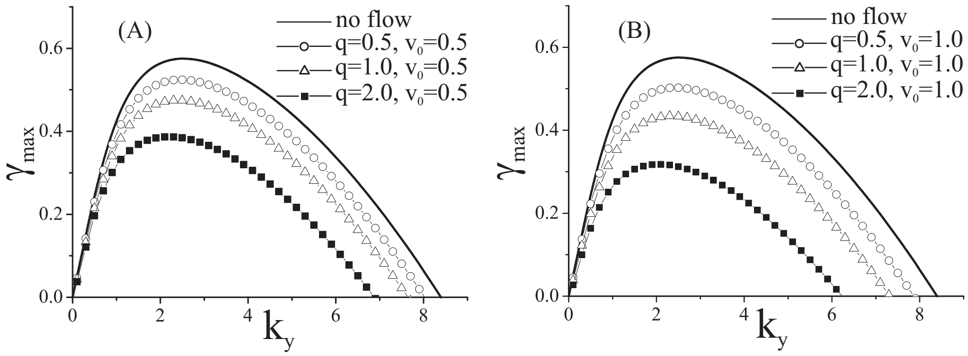

Figure 2. Maximum growth rate versus k_y for different values of q in equation (30) and (A) zonal flow amplitude $v_0 = 0.5$; (B) zonal flow amplitude $v_0 = 1$.

restricted to the region $k_{y,1} < k_y < k_{y,2}$. The lower $k_{y,1}$ and upper $k_{y,2}$ boundary values increase and decrease respectively as the root mean square zonal flow amplitude σ increases and above some critical value $\sigma_{cr} \sim 0.36$ (for $r = 0.25$) the instability of ETG modes is suppressed for all poloidal wave numbers k_y. An estimate for the critical value σ_{cr} can be obtained from equation (24). The growth rate remains zero as $k_y \to \infty$ (the most dangerous case) if $\sigma = \sigma_{cr}$. From equation (24) one can see that ω scales as k_y^{-1} as $k_y \to \infty$. Then, we can get the estimate $\sigma_{cr} = \sqrt{2}r$. This theoretical prediction for the dependence of the critical value of root mean square zonal flow amplitude on r is in very good agreement with numerical results.

Next, we consider the case when the zonal flow profile is deterministic and has the form $v(x) = v_0 \cos(qx)$. In addition, we include the effects of viscosity and thermal diffusivity. Then, equations (4) and (5) can be rewritten as an eigenvalue problem

$$\begin{pmatrix} k_y \hat{A}^{-1}\hat{B} & k_y \hat{A}^{-1} \\ -rk_y \hat{I} & k_y \hat{C} \end{pmatrix} \begin{pmatrix} \Phi \\ P \end{pmatrix} = \omega \begin{pmatrix} \Phi \\ P \end{pmatrix}, \tag{30}$$

where

$$\hat{A} = 1 + k_y^2 - \frac{d^2}{dx^2}, \quad \hat{B} = 1 + v_0 \cos(qx)\hat{A} - i\frac{\nu}{k_y}\hat{D},$$
$$\hat{D} = k_y^4 - 2k_y^2\frac{d^2}{dx^2} + \frac{d^4}{dx^4}, \quad \hat{C} = v_0 \cos(qx) - i\chi k_y + i\frac{\chi}{k_y}\frac{d^2}{dx^2}.$$

Employing a finite differencing approximation, we numerically solved the eigenvalue problem (30). The dissipative coefficients have been fixed at $\nu = \chi = 0.01$. The instability growth rate is plotted in figure 2 for two values of the zonal flow amplitude v_0 and different values of q (for $r = 0.5$). It is seen that the presence of zonal flow reduces the growth rate though the stabilizing effect manifests itself not so sharply as in the case of random shearing.

4. Conclusion

In conclusion, we have investigated the influence of zonal flows on the linear instability of ETG driven modes. Random and deterministic cos - like profiles of the zonal flow have been considered. For the random profile of zonal flow, we have obtained in the Bourrett approximation the dispersion relation for ETG modes in the presence of shearing. We have shown that the presence of random shearing caused by zonal flow has a strong stabilizing effect on the ETG driven mode destabilized by the temperature and pressure gradients. If the mean square amplitude of zonal flow exceeds some critical value, the linear instability of ETG modes is suppressed for all poloidal wave numbers k_y.

5. Acknowledgment

This work was supported by Ukranian Academy of Sciences through a programme 'Fundamental problems in particle physics and nuclear energy', grant number 150/166.

References

[1] Horton W, Hong B and Tang W 1988 *Phys. Fluids* **31** 2971
[2] Lee Y C, Doug P N, Guzdar P N and Liu C S 1987 *Phys. Fluids* **31** 1331
[3] Li J and Kishimoto Y 2002 *Phys. Plasmas* **9** 1241
[4] Jenko F, Dorland W,Kotschenreuther M and Rogers B N 2000 *Phys. Plasmas* **7** 1904
[5] Gürcan Ö D and Diamond P H 2004 *Phys. Plasmas* **11** 572
[6] Gürcan Ö D and Diamond P H 2004 *Phys. Plasmas* **11** 4973
[7] Diamond P H, Itoh S-I, Itoh K and Hahm T S 2005 *Plasma Phys. Control. Fusion* **47** R35
[8] Itoh K, Itoh S-I, Diamond P H, Hahm T S, Fujisawa A, Tynan G R, Yagi M and Nagashima Y 2006 *Phys. Plasmas* **13** 055502
[9] Smolyakov A I, Diamond P H and Malkov M 2000 *Phys. Rev. Lett.* **84** 491
[10] Smolyakov A I, Diamond P H and Shevchenko A I 2000 *Phys. Plasmas* **7** 1349
[11] Chen L, Lin Z and White R 2000 *Phys. Plasmas* **7** 3129
[12] Manfredi G, Roch C M and Dendy R O 2001 *Plasma Phys. Control. Fusion* **43** 825
[13] Zonca F, White R B and Chen L 2004 *Phys. Plasmas* **11** 2488
[14] Holland C and Diamond P H 2002 *Phys. Plasmas* **9** 3857
[15] Zaliznyak Yu A, Yakimenko A I and Lashkin V M 2008 *J. Plasma Phys.* **74** 381
[16] Tajima T, Horton W, Morrison P J, Schutkeker J, Kamimura T, Mima K and Abe Y 1991 *Phys. Fluids* B **3** 938
[17] Hamaguchi S and Horton W 1992 *Phys. Fluids* B **4** 319
[18] Waltz R E, Kerbel G D, Milovich J and Hammett G W 1995 *Phys. Plasmas* **2** 2408
[19] Sugama H and Wakatani M 1991 *Phys. Fluids* B **3** 1100
[20] Kim E and Diamond P H 2004 *Phys. Plasmas* **11** L77
[21] Krommes J A 1984 *Basic Plasma Physics II* ed A A Galeev and R N Sudan(Amsterdam: North -Holland)